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Intuitionistic Fuzzy Logic and the Probabilism of the Provisional Acceptance of a Scientific Theory

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1. Introduction

Media reports on climate change or empirical challenges to the accepted understanding of the nature and speed of light have demonstrated the inadequate understanding of experimental science in the popular mind, at a time when that same popular mind is being asked to make important bioethical decisions through their parliamentary representatives.

Much of the public debate, even by biomedical scientists, has been conducted without due regard to how the empirical sciences actually progress and even deliberate misunderstanding of the relation of evidence to conclusions [9]. For instance, it can be pointed out "to eager human embryo experimenters that nearly all the fundamental research can be done, and has not yet been done, on animal models, and that when it comes to clinical applications, not a single success so far has been achieved with embryonic stem cells – every success to date has been achieved by the use of adult cells" [17].

Most science, even mathematics, is conducted in a mode of 'conventionalism' [13], which involves provisional acceptance of hypotheses – the probabilism hinted at by Aquinas [12]. "The central question is one of partial belief, belief that may be quite pronounced, but which stops short of demonstrative or 'mathematical' certainty. Very simply, what kind of evidence and how much of it ought to be necessary to persuade a reasonable inquirer that it is more appropriate to accept a proposition than to reject it? How ought we to order degrees of belief that lie some-

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where between absolute conviction and utter dismissal? What degree of belief is necessary to justify an action with grave consequences?

This are not the sort of questions that mathematicians are given to worrying about, at least not when going about their mathematical business. But it is the central concern of 'practical reason', and in various forms it confronts us in many societal roles" [11]. Choy and Shannon elaborate this in the context of decision making in medicine [5]. It is the purpose of this note to examine the foundations of this provisional acceptance within the context of Intuitionistic fuzzy logic [2]. It is an extension of the work of Polikarov and Atanassov [13].

2. Scientific Explanation

The simplest explanation which fits the facts tends to be the prevailing confirmation in science. Scientists, being human, can be prone to disregard facts which do not fit this prevailing confirmation if their source is from authority less prestigious than the recognized authorities in their field. Argument from authority in science has historically hampered its progress.

Scientific progress is usually marked by 'confirmation' [4] or 'refutation' [15], although in practice the working scientist operates within a framework which contains a collection of hypotheses where there can be disagreement between empirical data and individual hypotheses without destroying the theory as a whole [6,10].

At this working stage of provisional acceptance, somewhere between refutation and confirmation, the empirical support of the theory prevails over any alleged counter-example. Such was Einstein's attitude when he said that "only after a more diverse body of observations becomes available will it be possible to decide with confidence whether the systematic deviations are due to a not yet recognized source of errors or to the circumstances that the foundations of the theory of relativity do not correspond to the facts."

3. Intuitionistic Fuzzy Logic

We shall now briefly outline the salient features of Intuitionistic Fuzzy Logic (IFL) by comparison with classical symbolic logic. IFL is a generalisation of the mathematical intuitionism of Brouwer [3,7] and the fuzzy sets of Zadeh [18].

In classical terms, to each proposition p, we assign a truth value denoted by *1* (truth) or *0* (falsity). In IFL we assign a truth value, $\mu(p)\ \varepsilon[0,1]$, for the degree of truth, and a falsity value, $v(p)\ \varepsilon[0,1]$ [1]:

$$0 \le \mu(p) + v(p) \le 1$$

This assignment is provided by an evaluation function V, which is defined over a set of propositions S,

$$V : S \to [0,1] \times [0,1]$$

such that

$$V(p) =< \mu(p), v(p) >$$

is an ordered pair. If the values V(p) and V(q) of the propositions p and q are known, then V can be extended:

$$\begin{gathered} V(\neg p) =< v(p), \mu(p) > \\ `V(p \wedge q) =< \min(\mu(p), \mu(q)), \max(v(p), v(q)) >, \\ V(p \vee q) =< \max(\mu(p), \mu(q)), \min(v(p), v(q)) >, \\ V(p \supset q) =< \max(v(p), \mu(q)), \min(\mu(p), v(q)) >; \end{gathered}$$

and, for the propositions $p, q \varepsilon S$:

$$\neg V(p) = V(\neg p),$$
$$V(p) \cap V(q) = V(p \wedge q),$$
$$V(p) \cup V(q) = V(p \vee q),$$
$$V(p) \to V(q) = V(p \supset q).$$

A *tautology* and an *intuitionistic fuzzy tautology* (IFT) are then defined respectively by

"A is a tautology" if, and only if, $V(A) = <1,0>$;

"A is an IFT" if, and only if, $V(A) = <a,b> \to a \geq b$.

4. Provisional Acceptance

Provisional acceptance of a scientific theory means that an individual counterexample of empirical evidence can be related to an individual hypothesis within an theoretical framework in order to modify some of the individual constituents of the theory and thus accommodate the disagreement. This can be written as

(a) $T_1 \equiv \big((A \supset C) \wedge \neg C\big) \supset \neg A,$
(b) $T_2 \equiv \big((A \wedge B \supset C) \wedge \neg C \wedge B\big) \supset \neg A,$
(c) $T_3 \equiv \big((A \wedge B \supset C) \wedge \neg C\big) \supset \big(\neg A \vee \neg B\big),$

for every three propositional forms *A*, *B* and *C*.

Theorem: T_1, T_2, T_3 are IFTs.
Proof: In the interests of brevity, we shall consider (b) only, as it is typical of all three parts.

$$\begin{aligned} V(T_2) &= \big[(<\mu_A,\nu_A> \wedge <\mu_B,\nu_B>) \to <\mu_C,\nu_C>\big] \wedge <\nu_C,\mu_C> \wedge <\mu_B,\nu_B> \to <\nu_A,\mu_A> \\ &= \big[<\max(\nu_A,\nu_B,\nu_C), \min(\mu_A,\mu_B,\mu_C)> \wedge <\min(\nu_C,\mu_B), \max(\mu_C,\nu_B)>\big] \to <\nu_A,\mu_A> \\ &= <\min(\nu_C,\mu_B), \max(\nu_A,\nu_B,\mu_C), \max(\mu_A,\mu_B,\nu_C)> \to <\nu_A,\mu_A> \\ &= <\max\big[\mu_c,\nu_B,\nu_A,\min(\mu_A,\mu_B,\nu_C)\big], \min\big[\nu_C,\mu_B,\mu_A,\max(\nu_A,\nu_B,\mu_C)\big]>, \end{aligned}$$

and

$$\begin{aligned} &\max\big[\mu_C,\nu_B,\nu_A,\min(\mu_A,\mu_B,\nu_C)\big] - \min\big[\nu_C,\mu_B,\mu_A,\max(\nu_A,\nu_B,\mu_C)\big] \\ &\geq \min(\mu_A,\mu_B,\nu_C) - \min(\nu_C,\mu_B,\mu_A) \\ &= 0. \end{aligned}$$

Therefore, *T* is an IFT. ■

5. Comments

The existence of an additional working *modus operandi* between refutation and confirmation can clarify the status of empirical scientific results. Furthermore, the schematic expression of this provisional acceptance of a theory invites an estimation of the truth values in any particular case so that the following type of analysis can be made.

Suppose that for the propositional forms *A* and *B*:

$V(A) \leq V(B)$ if, and only if, $(\mu_A \leq \mu_B) \wedge (\nu_A \leq \nu_B)$,

$V(A) > V(B)$ if, and only if, $(\mu_A > \mu_B) \wedge (\nu_A < \nu_B)$.

If we assume that μ_A, ν_A, the intuitionistic fuzzy values of *A* are fixed, then from the form of T_2 we see that T_2 is more reliable as the intuitionistic fuzzy truth of *B* increases, that is, the bigger μ_C and the smaller ν_C are.

The truth value of T_2 can also increase if any of

- $(V(A) > V(B)) \vee (V(A) > V(\neg C))$, for fixed μ_A;
- $(V(A) < V(B)) \vee (V(B) < V(\neg C))$, for fixed ν_A;
- $(V(A) < V(\neg C)) \vee (V(B) < V(\neg C))$, for fixed μ_B.

On the other hand, T_2 will not be changed if any of

- $(V(A) \leq V(B)) \vee (V(A) \leq V(\neg C))$, for fixed μ_A;
- $(V(A) \geq V(B)) \vee (V(B) \leq V(\neg C))$, for fixed ν_A;
- $(V(A) \geq V(\neg C)) \vee (V(B) \geq V(\neg C))$, for fixed μ_B.

Science should be no more exempt from moral evaluation than any other human activity, especially as it lacks the intellectual certitude of metaphysics and mathematics [16]. The logical analysis of 'provisional acceptance' will not make scientists more logical, but it is important that both scientists and the general public are aware of the nature and scope of science: "we cannot believe that what science knows is all there is" [9].

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